5 Arithmetic Progressions

Fastrack Revision

- Some numbers arranged in a definite order according to a definite rule, are said to form a sequence, where each number is called a term.
- A sequence whose terms follow a certain rule, is called a progression.
- A sequence in which each term differs from its preceding term by same constant except the first term is called an Arithmetic Progression (AP). This constant is called the common difference (d) of the AP. It can be positive, negative or zero.
- The general form of an AP is: $a, a+d, a+2d, \ldots$ where a is the first term.
- If a, b, c are three terms in AP, then $b-a=c-b$, i.e., $2b=a+c$.
- An AP with finite number of terms is a finite AP and which does not have finite number of terms is an infinite AP. Infinite AP's do not have a last term.
- In an AP, if we add, subtract, multiply or divide each term by the same non-zero number, then the resulting sequence would always be an AP.

Knowledge BOOSTER

1. *Three terms in AP should be taken as: $a-d$, a, $a+d$.*
2. *Four terms in AP should be taken as: $a-3d$, $a-d$, $a+d$, $a+3d$.*
3. *Five terms in AP should be taken as: $a-2d$, $a-d$, a, $a+d$, $a+2d$.*

- **General Term of an AP:** If the first term of an AP is a, common difference is d and its last term is l, then its nth or general term is given by $T_n=a_n=l=a+(n-1)d$.
- If nth term of an AP is a_n, then the common difference is determined by $d=a_n-a_{n-1}$.
- **nth Term from the End of an AP:** If the first term of an AP is a, its common difference is d and its last term is l, then nth term from the end $=l-(n-1)d$.

  And pth term from the end $=(n-p+1)$th term from the beginning.
- **Sum of n Terms of an AP:** If S_n is the sum of n terms of an AP, then $S_n=\frac{n}{2}[2a+(n-1)d]$ or $S_n=\frac{n}{2}[a+l]$

  where a is the first term and l is the last term.

Knowledge BOOSTER

1. *If sum of n terms (S_n) of an AP is given, then nth term (a_n) of an AP can be determined by $a_n=S_n-S_{n-1}$ and common difference $d=a_n-a_{n-1}$ $=S_n-2S_{n-1}+S_{n-2}$.*
2. **Arithmetic Mean (AM) between Two Numbers:** *If A is the AM between a and b, then*

$$A=\frac{a+b}{2}$$

Practice Exercise

Multiple Choice Questions

Q 1. 5th term of the sequence, whose nth term is $4n+2$, is:

a. 20 b. 22 c. 18 d. 23

Q 2. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is:

a. 1 b. $1/p$

c. -1 d. $-1/p$

Q 3. If $k+2$, $4k-6$ and $3k-2$ are three consecutive terms of an AP, then the value of k is: [CBSE 2023]

a. 3 b. -3

c. -4 d. -4

Q 4. The first four terms of an AP, whose first term is -2 and the common difference is -2, are: [NCERT EXEMPLAR]

a. $-2, 0, 2, 4$ b. $-2, 4, -8, 16$

c. $-2, -4, -6, -8$ d. $-2, -4, -8, -16$

Q 5. The next term of AP: $\sqrt{7}, \sqrt{28}, \sqrt{63}$ is: [CBSE 2023]

a. $\sqrt{70}$ b. $\sqrt{80}$

c. $\sqrt{97}$ d. $\sqrt{112}$

Q 6. The next two terms of an AP sequence 5, 8, 11, ... are:

a. 13, 16 b. 14, 17 c. 15, 18 d. 12, 15

Q 7. In an AP, if $a = 5, d = 3$ and $n = 10$, then the value of a_{10} is:

a. 31 b. 32 c. 34 d. 36

Q 8. The number of terms of an AP, having first term 5, common difference 3 and last term 74, is:

a. 23 b. 24 c. 25 d. 26

Q 9. The nth term of the AP: $a, 3a, 5a, ...$ is: *[CBSE 2020]*

a. na b. $(2n-1)a$

c. $(2n+1)a$ d. $2na$

Q 10. Which term of the AP: 21, 42, 63, 84,... is 210? *[NCERT EXEMPLAR]*

a. 9th b. 10th

c. 11th d. 12th

Q 11. The 21st term of the AP, whose first two terms are –3 and 4, is: *[NCERT EXEMPLAR]*

a. 17 b. 137 c. 143 d. –143

Q 12. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$? *[NCERT EXEMPLAR]*

a. 5 b. 20 c. 25 d. 30

Q 13. The 7th term from the end of the AP: $17, 14, 11, ..., -40$ is:

a. – 18 b. – 22 c. – 25 d. – 20

Q 14. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term? *[NCERT EXEMPLAR]*

a. 30 b. 33 c. 37 d. 38

Q 15. Two AP's have the same common difference. The first term of one of these is –1 and that of the other is –8. Then the difference between their 4th term is: *[CBSE SQP 2023-24]*

a. 1 b. –7 c. 7 d. 9

Q 16. If the sum of the first n terms of an AP be $3n^2 + n$ and its common difference is 6, then its first term is: *[CBSE 2023]*

a. 2 b. 3 c. 1 d. 4

Q 17. If the sum of n terms of an AP is $S_n = 2n^2 + 3$, then common difference of an AP is:

a. 3 b. 4 c. 2 d. – 2

Q 18. The sum of first 7 terms of an AP: $2, 4, 6, 8, 10, ...$ is:

a. 52 b. 54 c. 56 d. 58

Q 19. In an AP, if $a = 1, a_n = 20$ and $S_n = 399$, then n is equal to:

a. 38 b. 39 c. 40 d. 41

Q 20. The number of terms of an AP: $64, 60, 56, ...$, whose sum is 544, is:

a. 15, 16 b. 16, 17 c. 14, 15 d. 17, 18

Assertion & Reason Type Questions

Directions (Q. Nos. 21-27): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 21. Assertion (A): $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$ is in Arithmetic Progression.

Reason (R): The terms of an Arithmetic Progression cannot have both positive and negative rational numbers. *[CBSE SQP 2023-24]*

Q 22. Assertion (A): The nth term of the sequence $-8, -4, 0, 4, ...$ is $4n - 12$.

Reason (R): The nth term of an AP is determined by $T_n = a + (n-1)d$.

Q 23. Assertion (A): The common difference of an AP in which $a_{20} - a_{16} = 20$ is 5

Reason (R): The nth term of the sequence $\sqrt{2}, \sqrt{4}, \sqrt{18}, ...$ is $\sqrt{2}\,n$.

Q 24. Assertion (A): a, b, c are in AP if and only if $2b = a + c$.

Reason (R): The sum of first n odd natural numbers is n^2. *[CBSE 2023]*

Q 25. Assertion (A): The sum of first 20 even natural numbers divisible by 5 is 2110.

Reason (R): The sum of n terms of an AP is given by $S_n = \frac{n}{2}(a + l)$, where l is the last term of an AP.

Q 26. Assertion (A): The sum of the series with the nth term $T_n = 7 - 3n$ is –255, when number of terms is $n = 15$.

Reason (R): The sum of AP series is determined by $S_n = \frac{n}{2}[2a + (n-1)d]$.

Q 27. Assertion (A): If sum of first n terms of an AP is $S_n = 6n^2 - 2n$, then nth term of an AP is $12n - 8$.

Reason (R): Suppose S_n be the sum of n terms of an AP, then nth term of an AP is $T_n = S_{n-1} - S_n$.

Fill in the Blanks Type Questions

Q 28. If the common difference is ...., then each term of the AP will be same as the first term of the AP.

Q 29. The value of k for which $k^2 + 4k + 8, 2k^2 + 3k + 6, 3k^2 + 4k + 4$ are three consecutive terms of an AP, is .... *[NCERT EXEMPLAR]*

Q 30. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be ....

Q 31. .... term of the AP: 24, 21, 18, 15, ... is the first negative term.

Q 32. The sum of 10 terms of an AP: $-8, -6, -4, ...$ is ....

True/False Type Questions

Q 33. If we multiply each term of an AP by 2, then the resulting sequence is an AP.

Q 34. In an AP, if $a = 1, a_n = 20$ and $n = 38$, then sum of first 38 terms is 499.

Q 35. If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is 5.

Q 36. 20 terms of AP: 18, 16, 14, ... should be taken so that their sum is zero.

Q 37. If sum of the first n terms of an AP is given by $S_n = 3n^2 + 4$, then its nth term is $6n - 3$.

Solutions

1. (b) Given, $a_n = 4n + 2$

Put $n = 5$, we get

$$a_5 = 4(5) + 2 = 20 + 2 = 22$$

2. (c) Common difference

$$= \frac{1-p}{p} - \frac{1}{p}$$

$$= \frac{1}{p} - 1 - \frac{1}{p} = -1$$

3. (a) Given terms of an AP are

$k + 2, 4k - 6$ and $3k - 2$.

Therefore, common difference of each term should be equal.

So, $(4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$\Rightarrow \quad 3k - 8 = -k + 4$

$\Rightarrow \quad 4k = 12 \Rightarrow k = 3$

4. (c) Let a be the first term and d be the common difference of an AP.

Then $a = -2$ and $d = -2$

First four terms of an AP are

$a, a + d, a + 2d, a + 3d$.

Here, $a = -2$

$a + d = -2 - 2 = -4$

$a + 2d = -2 + 2(-2) = -2 - 4 = -6$

and $a + 3d = -2 + 3(-2) = -2 - 6 = -8$

Hence, first four terms of an AP are –2, –4, –6, –8.

5. (d) Given AP: $\sqrt{7}, \sqrt{28}, \sqrt{63}, ...$

Here, $a_1 = \sqrt{7}, a_2 = \sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$

and $a_3 = \sqrt{63} = \sqrt{9 \times 7} = 3\sqrt{7}$

$\therefore$ Common difference $(d) = a_2 - a_1 = 2\sqrt{7} - \sqrt{7} = \sqrt{7}$

So, next term *i.e.*, 4th term of AP is

$a_4 = a_1 + (4 - 1)d \quad [\because a_n = a + (n - 1)d]$

$\Rightarrow \quad a_4 = \sqrt{7} + 3\sqrt{7} = 4\sqrt{7}$.

$= \sqrt{4^2 \times 7} = \sqrt{16 \times 7} = \sqrt{112}$

6. (b) Given AP : 5, 8, 11,....

Here, $a_1 = 5, a_2 = 8$ and $a_3 = 11$

$\therefore$ Common difference $(d) = a_2 - a_1 = 8 - 5 = 3$

So, next two terms *i.e.*, 4th and 5th terms of AP are

$a_4 = a_1 + (4 - 1)d = 5 + 3 \times 3 = 5 + 9 = 14$

and $a_5 = a_1 + (5 - 1)d = 5 + 4 \times 3 = 5 + 12 = 17$

7. (b) Given, $a = 5, d = 3$ and $n = 10$

$\because a_n = a + (n - 1)d$

$\therefore a_{10} = 5 + (10 - 1) \times 3 = 5 + 27 = 32$

8. (b) Given, $a = 5, d = 3$

and last term $(l) = 74$

$\because \quad l = a + (n - 1)d$

$74 = 5 + (n - 1)(3)$

$\Rightarrow \quad n - 1 = \frac{69}{3} = 23$

$\therefore \quad n = 24$

So, the number of terms is 24.

9. (b) Given, first term $(A) = a$,

common difference $(D) = 3a - a = 2a$

$\because$ nth term of AP,

$a_n = A + (n - 1)D$

$\therefore a_n = a + (n - 1)2a = a + 2an - 2a$

$= 2an - a = (2n - 1)a$

10. (b) Given AP sequence is 21, 42, 63, 84, ...

Here, first term, $a = 21$

Common difference, $d = 42 - 21 = 21$

and last term, $l = 210$

Then, nth term of an AP sequence is given by

$T_n = l = a + (n - 1)d$

$\therefore \quad 210 = 21 + (n - 1)21$

$\Rightarrow \quad 210 = 21 + 21n - 21$

$\Rightarrow \quad 21n = 210$

$\Rightarrow \quad n = 10$

Hence, 10th term of an AP is 210.

11. (b) Given, $a_1 = -3$ and $a_2 = 4$

Here, common difference, $d = a_2 - a_1 = 4 - (-3) = 7$

and first term, $a = -3$

Then nth term of an AP is given by

$T_n = a + (n - 1)d$

Therefore, 21st term of an AP is

$T_{21} = -3 + (21 - 1)(7)$

$= -3 + 20 \times 7 = -3 + 140 = 137$

12. (c) Let a be the first term and d be the common difference of the given AP.

Given, $d = 5$

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nth term of an AP is given by

$a_n = a + (n - 1)d$

$\therefore \quad a_{18} - a_{13} = a + (18 - 1)d - [a + (13 - 1)d]$

$= 17d - 12d = 5d = 5 \times 5 = 25$

13. (b) Given AP is: 17, 14, 11, ..., – 40
Here, first term, $a = 17$,
Common difference, $d = 14 - 17 = -3$
and last term, $l = -40$

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> *nth term from the end of an AP is $l - (n-1)d$.*

∴ 7th term from the end of an AP
$= -40 - (7-1)(-3)$
$= -40 - 6(-3) = -40 + 18 = -22$

14. (b) Let a and d be the first term and common difference of an AP respectively.
Then, nth term of an AP is
$$T_n = a + (n-1)d$$
It is given that
$$T_2 = 13$$
and $T_5 = 25$
∴ $a + (2-1)d = 13$
⇒ $a + d = 13$ ...(1)
and $a + (5-1)d = 25$
⇒ $a + 4d = 25$ ...(2)
Subtract eq. (1) from eq. (2), we get
$(a + 4d) - (a + d) = 25 - 13$
⇒ $3d = 12 \Rightarrow d = 4$
Put $d = 4$ in eq. (1), we get
$a + 4 = 13 \Rightarrow a = 9$
∴ 7th term of an AP is
$$T_7 = 9 + (7-1)4 = 9 + 6 \times 4 = 9 + 24 = 33$$

15. (c) Let first term of first AP and second AP be a and a' respectively. The same common difference of both AP is d.
Given that, $a = -1$ and $a' = -8$
∴ 4th term of first AP, $a_4 = a + (4-1)d = -1 + 3d$
and 4th term of second AP, $a'_4 = a' + (4-1)d$
$= -8 + 3d$
So, difference between 4th terms of both AP
$= a_4 - a'_4$
$= (-1 + 3d) - (-8 + 3d) = -1 + 3d + 8 - 3d = 7$

16. (d) Given, sum of first 'n' terms of an AP,
$S_n = 3n^2 + n$
Then, nth term of an AP is determined by
$a_n = S_n - S_{n-1} = (3n^2 + n) - 3(n-1)^2 - (n-1)$
$= 3n^2 + n - 3(n^2 + 1 - 2n) - (n-1)$
$= 3n^2 + n - 3n^2 - 3 + 6n - n + 1$
$= 6n - 2$
∴ First term of an AP is $a_1 = 6 \times 1 - 2$ (put $n = 1$)
$= 6 - 2 = 4$

17. (b) Given, $S_n = 2n^2 + 3$.
Then, nth term of an AP is determined by
$a_n = S_n - S_{n-1} = (2n^2 + 3) - [2(n-1)^2 + 3]$
$= (2n^2 + 3) - [2n^2 - 4n + 5] = 4n - 2$
Now, the common difference of an AP is given by
$d = a_n - a_{n-1} = 4n - 2 - [4(n-1) - 2]$
$= 4n - 2 - 4n + 4 + 2 = 4$

18. (c) Given sequence of an AP is 2, 4, 6, 8, 10, ...
Here, first term, $a = 2$,
Common difference, $d = 4 - 2 = 2$
The sum of first n terms of an AP is given by
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
∴ The sum of first 7 terms of an AP is
$$S_7 = \frac{7}{2}[2 \times 2 + (7-1)2] = \frac{7}{2} \times 2[2 + (7-1)]$$
$$= 7[2 + 6] = 7 \times 8 = 56$$

19. (a) Given, $a = 1$, $a_n = 20$ and $S_n = 399$.
The sum of n terms of an AP is given by
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
∴ $399 = \frac{n}{2}[2 \times 1 + (n-1)d]$
⇒ $798 = 2n + n(n-1)d$ ...(1)
Also, $a_n = 20$
∴ $a + (n-1)d = 20$
⇒ $1 + (n-1)d = 20$
⇒ $(n-1)d = 19$
Put $(n-1)d = 19$ in eq. (1), we get
$798 = 2n + 19n$
⇒ $798 = 21n \Rightarrow n = 38$

20. (b) Given AP is 64, 60, 56, ...
Here, $a = 64$, $d = 60 - 64 = -4$
Let n be the number of terms in the given AP.
Then, $S_n = 544$
∴ $\frac{n}{2}[2a + (n-1)d] = 544$
⇒ $\frac{n}{2}[2 \times 64 + (n-1)(-4)] = 544$
⇒ $\frac{n}{2} \times 2[64 - 2(n-1)] = 544$
⇒ $n[66 - 2n] = 544$
⇒ $2n^2 - 66n + 544 = 0$
⇒ $n^2 - 33n + 272 = 0$
⇒ $n^2 - (17 + 16)n + 272 = 0$
(by splitting the middle term)
⇒ $n^2 - 17n - 16n + 272 = 0$
⇒ $n(n-17) - 16(n-17) = 0$
⇒ $(n-16)(n-17) = 0$
⇒ $n - 16 = 0$ or $n - 17 = 0$
⇒ $n = 16$ or $n = 17$

21. (c) **Assertion (A):** Given sequence: $-5, -\frac{5}{2}, 0, \frac{5}{2}, ...$
Here, $a_1 = -5, a_2 = -\frac{5}{2}, a_3 = 0, a_4 = \frac{5}{2}$
Difference of two consecutive terms:
$$a_2 - a_1 = \frac{-5}{2} - (-5) = -\frac{5}{2} + 5 = \frac{5}{2}$$
$$a_3 - a_2 = 0 - \left(-\frac{5}{2}\right) = \frac{5}{2}$$
$$a_4 - a_3 = \frac{5}{2} - 0 = \frac{5}{2}$$

Since, the difference of two consecutive terms is constant *i.e.*, $\frac{5}{2}$.

Therefore, given sequence is an AP.

So, Assertion (A) is true.

Reason (R): The terms of an AP. can have both positive and negative rational numbers.

So, Reason (R) is false.

Hence, Assertion (A) is true but Reason (R) is false.

22. (a) **Assertion (A):** Given sequence is $-8, -4, 0, 4, \ldots$

$\because \quad a_2 - a_1 = -4-(-8) = 4,$

$a_3 - a_2 = 0-(-4) = 4,$

$a_4 - a_3 = 4 - 0 = 4$

Here, we see that difference of two consecutive terms is same constant. So, given sequence is an AP.

$\therefore$ First term, $a = -8$

and common difference, $d = 4$

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nth term of an AP is

$$T_n = a + (n-1)d$$

$\therefore \quad T_n = -8 + (n-1)(4)$

$= -8 + 4n - 4 = 4n - 12$

So, Assertion (A) is true.

Reason (R): It is also true that *n*th term of an AP is determined by $T_n = a + (n-1)d$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

23. (b) **Assertion (A):** Let a and d be the first term and common difference of an AP. Then *n*th term of an AP is

$$a_n = a + (n-1)d$$

Given, $a_{20} - a_{16} = 20$

$\therefore [a+(20-1)d] - [a+(16-1)d] = 20$

$19d - 15d = 20$

$\Rightarrow \quad 4d = 20$

$\Rightarrow \quad d = 5$

So, Assertion (A) is true.

Reason (R): Given sequence is

$\sqrt{2}, \sqrt{4}, \sqrt{18}, \ldots$

or $\quad \sqrt{2}, 2\sqrt{2}, 3\sqrt{2} \ldots$

Here $a = \sqrt{2}, d = 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$\because \quad T_n = a + (n-1)d$

$\therefore \quad T_n = \sqrt{2} + (n-1)\sqrt{2} = \sqrt{2}n$

So, Reason (R) is also true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

24. (b) **Assertion (A):**

If part: Given a, b, c are in AP.

Then $\quad b - a = c - b$

$\Rightarrow \quad b + b = a - c \Rightarrow 2b = a + c$

Only part: Given, $2b = a + c$

$\Rightarrow \quad b + b = a + c$

$\Rightarrow \quad b - a = c - b$

$\Rightarrow \quad a_2 - a_1 = a_3 - a_2 \quad$ (let $a_1 = a, a_2 = b$ and $a_3 = c$)

Since, each term differs from its preceding term are equal.

$\therefore$ The sequence a_1, a_2, a_3 or a, b, c are in AP.

Therefore, a, b, c are in AP if and only if $2b = a + c$.

So, Assertion (A) is true.

Reason (R): First n odd natural numbers are: $1, 3, 5, 7, \ldots$

Here, first term $(a) = 1$

and common difference $(d) = 3 - 1 = 5 - 3 = 2$

Since, the difference between each consecutive terms is constant *i.e.*, 2.

So, the sequence forms an AP.

$\therefore$ Sum of first n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2\times 1 + (n-1)\times 2]$$

$$= \frac{n}{2}\times 2(1 + n - 1) = n \cdot n = n^2$$

So, Reason (R) is true.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

25. (d) **Assertion (A):** Even natural numbers divisible by 5 are 10, 20, 30, 40, ...

They form an AP with first term, $a = 10$

and common difference, $d = 20 - 10 = 30 - 20 = 10$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{20} = \frac{20}{2}[2\times 10 + (20-1)10]$$

$$= 10[20 + 190] = 2100$$

So, Assertion (A) is false.

Reason (R): It is true that the sum of n terms of an AP is $S_n = \frac{n}{2}(a + l)$.

So, Reason (R) is true.

Hence, Assertion (A) is false but Reason (R) is true.

26. (a) **Assertion (A):** Given, *n*th term is $T_n = 7 - 3n$.

Here, $\quad T_1 = 7 - 3(1) = 4$

$T_2 = 7 - 3(2) = 1$

$T_3 = 7 - 3(3) = -2$

Here, sequence is $4, 1, -2, \ldots$

Now, $\quad 1 - 4 = -3, \; -2 - 1 = -3$

Here, difference of two consecutive terms is same. So, it is an A.P.

Here, first term $a = 4$ and common difference $d = -3$

$\therefore$ The sum of 15 terms of an AP is

$$S_{15} = \frac{15}{2}[2\times 4 + (15-1)(-3)] \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= \frac{15}{2}[8 - 42] = \frac{15}{2}\times(-34) = -255$$

So, Assertion (A) is true.

Reason (R): It is also true that sum of n terms of an AP is determined by $S_n = \frac{n}{2}[2a + (n-1)d]$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

27. (c) **Assertion (A):** Given, $S_n = 6n^2 - 2n$.

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nth term of an AP, whose sum is S_n, is

$$T_n = S_n - S_{n-1}$$

Using formula,

$$T_n = S_n - S_{n-1} = (6n^2 - 2n) - [6(n-1)^2 - 2(n-1)]$$
$$= 6n^2 - 2n - [6(n^2 + 1 - 2n) - 2n + 2]$$
$$= 6n^2 - 2n - [6n^2 - 14n + 8]$$
$$= -2n + 14n - 8 = 12n - 8$$

So, Assertion (A) is true.

Reason (R): It is not true that

$$T_n = S_{n-1} - S_n$$

Thus, the correct relation is

$$T_n = S_n - S_{n-1}$$

Hence, Assertion (A) is true but Reason (R) is false.

28. Zero

29. Given, three consecutive terms of an AP are $k^2 + 4k + 8$, $2k^2 + 3k + 6$ and $3k^2 + 4k + 4$.

Therefore, common difference of each term of an AP is equal.

Here, $a_1 = k^2 + 4k + 8$, $a_2 = 2k^2 + 3k + 6$

and $a_3 = 3k^2 + 4k + 4$

$\therefore$ $a_2 - a_1 = a_3 - a_2$

$\Rightarrow$ $(2k^2 + 3k + 6) - (k^2 + 4k + 8)$
$= (3k^2 + 4k + 4) - (2k^2 + 3k + 6)$

$\Rightarrow$ $k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow$ $2k = 0 \Rightarrow k = 0$

30. Let a be the first term and d be the common difference of an AP. Then according to the given condition,

$$7\,T_7 = 11\,T_{11}$$

$\therefore$ $7\,[a + (7-1)d] = 11\,[a + (11-1)d]$ $[\because T_n = a + (n-1)d]$

$\Rightarrow$ $7[a + 6d] = 11\,[a + 10d]$

$\Rightarrow$ $7a + 42d = 11a + 110d$

$\Rightarrow$ $4a + 68d = 0$

$\Rightarrow$ $a + 17d = 0$ ...(1)

Now, the 18th term of an AP is

$$T_{18} = a + (18-1)d$$
$= a + 17d = 0$ [from eq. (1)]

31. Let nth term of the AP : 24, 21, 18, 15, ... be the first negative term.

Here, first term $(a) = 24$

and common difference $(d) = 21 - 24 = -3$

$\because$ $a_n = a + (n-1)d$

$\therefore$ $[a + (n-1)d] < 0$

$\Rightarrow$ $[24 + (n-1)(-3)] < 0$

$\Rightarrow$ $(24 - 3n + 3) < 0$

$\Rightarrow$ $27 - 3n < 0$

$\Rightarrow$ $3n > 27 \Rightarrow n > 9$

$\therefore$ $n = 10$

So, 10th term is the first negative term.

COMMON ERR!R

Sometimes students take the value of $n = 9$ instead of taken the value of $n = 10$.

32. Given AP is $-8, -6, -4,$ ....

Here, first term, $a = -8$

and common difference, $d = -6 - (-8) = 2$

Then, sum of an AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times (-8) + (10-1)(2)]$$
$$= 5[-16 + 18] = 5 \times 2 = 10$$

33. True

34. Given, $a = 1$, $a_n = 20$ and $n = 38$

$\because$ $S_n = \frac{n}{2}[a + a_n]$

$$S_{38} = \frac{38}{2}[1 + 20]$$

$\Rightarrow$ $S_{38} = 19 \times 21$

$\Rightarrow$ $S_{38} = 399$

Hence, given statement is false.

35. Let a and d be the first term and common difference respectively. Then

$a = -5$ and $d = 2$

The sum of first n terms of an AP is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_6 = \frac{6}{2}[2 \times (-5) + (6-1)2]$$
$$= 3[-10 + 10] = 0$$

Hence, given statement is false.

36. Given, AP is 18, 16, 14, ...

Here, $a = 18$, $d = 16 - 18 = -2$

Now, sum of 20 terms of an AP is

$$S_{20} = \frac{20}{2}[2 \times 18 + (20-1)(-2)]$$
$$\left[\because S_n = \frac{n}{2}(2a + (n-1)d)\right]$$
$$= 10[36 - 38] = 10 \times (-2) = -20$$

Hence, given statement is false.

37. Given, $S_n = 3n^2 + 4$

$\therefore$ nth term, $a_n = S_n - S_{(n-1)}$

$$= 3n^2 + 4 - [3(n-1)^2 + 4]$$
$$= 3n^2 + 4 - [3(n^2 + 1 - 2n) + 4]$$
$$= 3n^2 + 4 - 3n^2 + 6n - 7$$
$$= 6n - 3$$

Hence, given statement is true.

Case Study Based Questions

Case Study 1

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production. The production of air conditioner in a factory increases uniformly by a fixed number every year. It produced 12000 sets in 3rd year and 20400 in 10th year.

Based on the above information, solve the following questions:

Q 1. Find the production during first year.

a. 9600 b. 1200 c. 2400 d. 8000

Q 2. Find the production during 8th year.

a. 9600 b. 18000
c. 8000 d. 14400

Q 3. Find the production during first five years.

a. 55400 b. 50800
c. 60000 d. 62600

Q 4. In which year, the production is 30000?

a. 15 b. 16 c. 17 d. 18

Q 5. Find the difference of the production during 7th year and 5th year.

a. 2400 b. 1200 c. 9600 d. 4000

Solutions

1. Given that, the production of air conditioner in a factory increases uniformly by a fixed number every year, *i.e.* production of air conditioner in every year form an AP.

Let the first term and common difference of this AP be 'a' and 'd' respectively.

According to the question, the factory produced 12000 air conditioner in 3rd year.

TR!CK

nth term of an AP is $T_n = a + (n-1)d$
where, a and d are first term and common difference respectively.

i.e. $T_3 = a + (3-1)d$

$\Rightarrow$ $12000 = a + 2d$ ...(1)

and 20400 air conditioner in 10th year.

i.e. $T_{10} = a + (10-1)d$

$\Rightarrow$ $20400 = a + 9d$ ...(2)

Now subtracting eq. (1) from eq. (2), we get

$(a+9d) - (a+2d) = 20400 - 12000$

$\Rightarrow$ $7d = 8400$

$\therefore$ $d = \dfrac{8400}{7} = 1200$

Put the value of d in eq. (1), we get

$12{,}000 = a + 2 \times 1200$

$\Rightarrow$ $12{,}000 = a + 2400$

$\therefore$ $a = 12000 - 2400$

$= 9600$

Hence, the production during first year is 9600.

So, option (a) is correct.

2. The production during 8th year is

$T_8 = a + (8-1)d$
$= 9600 + 7 \times 1200$
$= 9600 + 8400$
$= 18000$

So, option (b) is correct.

3.

TR!CK

The sum of n terms of an AP is $S_n = \dfrac{n}{2}[2a + (n-1)d]$

The production during first five years is

$S_5 = \dfrac{5}{2}(2 \times 9600 + (5-1)1200)$

$= \dfrac{5}{2}(19200 + 4 \times 1200)$

$= \dfrac{5}{2}(19200 + 4800)$

$= \dfrac{5}{2} \times 24000$

$= 5 \times 12000 = 60000$

So, option (c) is correct.

4. Let nth year, the production is

$T_n = 30000$

$\Rightarrow a + (n-1)d = 30000$ $[\because T_n = a + (n-1)d]$

$\Rightarrow 9600 + (n-1)1200 = 30000$

$\Rightarrow$ $n - 1 = \dfrac{20400}{1200}$

$\therefore$ $n = 17 + 1 = 18$

Hence, in 18th year, the production is 30000.

So, option (d) is correct.

5. Now, the production during 7th year is

$T_7 = a + (7-1)d$ $[\because T_n = a + (n-1)d]$
$= 9600 + 6 \times 1200$
$= 9600 + 7200$
$= 16800$

and the production during 5th year is

$T_5 = a + (5-1)d$
$= 9600 + 4 \times 1200$
$= 9600 + 4800 = 14400$

∴ The difference of the production during 7th year and 5th year = 16800 – 14400 = 2400

So, option (a) is correct.

Case Study 2

Your younger sister wants to buy an electric car and plans to take loan from a bank for her electric car. She repays her total loan of ₹ 321600 by paying every month starting with the first instalment of ₹ 2000 and it increases the instalment by ₹ 200 every month.

Based on the above information, solve the following questions:

Q 1. Find the list of the instalment formed by the given statement.

a. 2000, 1800, 1600,... b. 2000, 2200, 2400,... c. 2200, 2400, 2600,... d. 2300, 2600, 2900,...

Q 2. The amount paid by her in 25th instalment is:

a. ₹ 6800 b. ₹ 3500 c. ₹ 4800 d. ₹ 6600

Q 3. Find the difference of the amount in 4th and 6th instalment paid by younger sister.

a. ₹ 200 b. ₹ 400 c. ₹ 600 d. ₹ 800

Q 4. In how many instalment, she clear her total bank loan?

a. 1582 b. 1580 c. 1599 d. 1600

Q 5. Find the sum of the first seven instalments.

a. ₹ 14000 b. ₹ 13600 c. ₹ 10400 d. ₹ 12600

Solutions

1. It can be observed that these instalments are in AP having first term (instalment) as ₹ 2000 and common difference (increase instalment) as ₹200.

Here, $a = 2000$ and $d = 200$

Therefore list of an AP is $a, a+d, a+2d, \ldots$

i.e., $2000, 2000+200, 2000+2\times 200, \ldots$

i.e., $2000, 2200, 2400, \ldots$

So, option (b) is correct.

2. It can be observed that these instalments are in an AP having first term (instalment) as ₹ 2000 and common difference (increase instalment) as ₹ 200.

Here, $a = 2000$ and $d = 200$

TR!CK

nth term of an AP is, $T_n = a + (n-1)d$

where, a and d are first term and common difference respectively.

∴ The amount paid by her in 25th instalment is

$T_{25} = a + (25-1)d$

$= 2000 + 24 \times 200$

$= 2000 + 4800 = ₹\, 6800$

So, option (a) is correct.

3. Let a and d be the first term and common difference of an AP.

Then, $a_4 = a + (4-1)d$ $[\because a_n = a + (n-1)d]$

$= a + 3d$

Similarly, $a_6 = a + 5d$.

∴ Required difference $= a_6 - a_4$

$= (a+5d) - (a+3d) = 2d$

$= 2 \times 200 = ₹\, 400$

So, option (b) is correct.

4. Let in n instalments, she clear her loan.

Given, $T_n = 321600$

$\because$ $T_n = a + (n-1)d$

$\therefore$ $321600 = 2000 + (n-1)200$

$\Rightarrow 319600 = (n-1)200$

$\Rightarrow 1598 = n - 1$

$\Rightarrow n = 1599$

So, in 1599 instalments, she clear her bank loan.

So, option (c) is correct.

5. Here, $a = 1200$, $d = 200$

∴ The sum of first seven instalments is

$$S_7 = \frac{7}{2}[2 \times 1200 + (7-1)200]$$

$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$

$$= \frac{7}{2}[2400 + 1200]$$

$$= \frac{7}{2}[3600] = 7 \times 1800 = ₹12600$$

So, option (d) is correct.

Case Study 3

In an examination hall, the examiner makes students sit in such a way that no students can cheat from other student and make no student sit uncomfortably. So, the teacher decides to mark the numbers on each chair from 1, 2, 3, ...

There are 25 students and each student is seated at alternate position in examination room such that the sequence formed is 1, 3, 5, ...

Based on the above information, solve the following questions:

Q 1. What type of sequence is formed, to follow the seating arrangement of students in the examination room?

Q 2. Find the seat number of the last student in the examination room.

Q 3. Find the seat number of 10th vacant seat in the examination room.

Solutions

1. Given, seating arrangement of students in the examination room is 1, 3, 5, ...

   Here, $a_1 = 1, a_2 = 3, a_3 = 5, ...$

   Now, $a_2 - a_1 = 3 - 1 = 2$

   $a_3 - a_2 = 5 - 3 = 2$

   Here, common difference is same, so given sequence is the type of Arithmetic progression.

2. Given, $a = 1, d = 3 - 1 = 2$ and $n = 25$

   $\because \quad T_n = a + (n-1)d$

   There are 25 students.

   $\therefore \quad T_{25} = 1 + (25-1)2 = 1 + 24 \times 2 = 49$

   Hence, last student will sit on the 49th seat number.

3. The sequence of vacant seats are as follows, 2, 4, 6, ..., 48.

   Here, $a = 2, d = 4 - 2 = 6 - 4 = 2$

   The 10th vacant seat will be

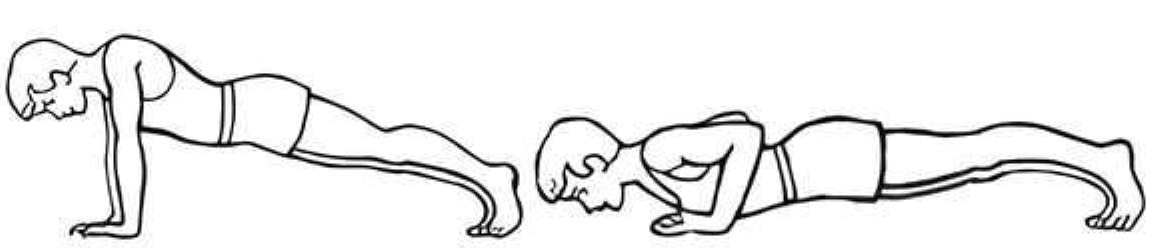
   $T_{10} = a + (10-1)d \quad [\because T_n = a + (n-1)d]$

   $= 2 + 9 \times 2 = 2 + 18 = 20$

   Hence, the 10th vacant seat number is 20.

Case Study 4

Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms and shoulders, support required from other muscles helps in toning up the whole body.



Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved.

Based on the above information, solve the following questions: [CBSE 2022 Term-II]

Q 1. Form an AP representing the number of push-ups per day and hence find the minimum number of days he needs to practice before the day his goal is accomplished.

Q 2. Find the total number of push-ups performed by Nitesh up to the day his goal is achieved.

Solutions

1. In first day, Nitesh makes 3000 push-ups and he is increasing 5 push-ups each day.

   Therefore first term, $a = 3000$

   and common difference, $d = 5$

   AP sequence is $a, a+d, a+2d, a+3d, ...$

   $\therefore$ 3000, $3000 + 5, 3000 + 2 \times 5, 3000 + 3 \times 5, ...$

   or 3000, 3005, 3010, 3015, ...

   It is given that $a_n = 3900$

   $\because \quad a_n = a + (n-1)d$

   $\therefore \quad 3900 = 3000 + (n-1)5$

   $\Rightarrow \quad (n-1)5 = 900$

   $\Rightarrow \quad n - 1 = 180$

   $\Rightarrow \quad n = 181$

   Hence, minimum number of days he needs to practice before the day his goal accomplished is 181.

2. The total number of push-ups performed by Nitesh to the day his goal achieved is

   $$S_n = \frac{n}{2}[a + a_n]$$

   $$= \frac{181}{2}[3000 + 3900]$$

   $$= \frac{181}{2} \times 6900$$

   $$= 181 \times 3450 = 624450$$

Case Study 5

The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

Based on the above information, solve the following questions: [CBSE SQP 2022-23]

Q 1. If the first circular row has 30 seats, how many seats will be there in the 10th row?

Q 2. For 1500 seats in the auditorium, how many rows need to be there?

Or

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?

Q 3. If there were 17 rows in the auditorium, how many seats will be there in the middle row?

Solutions

1. Since, each row is increasing by 10 seats, so it is an AP with first term $a = 30$ and common difference $d = 10$.

So, number of seats in 10th row $= a_{10} = a + 9d$

$[\because a_n = a + (n-1)d]$

$= 30 + 9 \times 10 = 120$

2.

TR!CK

The sum of n terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$

$$1500 = \frac{n}{2}[2 \times 30 + (n-1)10]$$

$\Rightarrow \quad 3000 = 50n + 10n^2$

$\Rightarrow \quad n^2 + 5n - 300 = 0$

$\Rightarrow \quad n^2 + 20n - 15n - 300 = 0$

(by splitting the middle term)

$\Rightarrow \quad n(n+20) - 15(n+20) = 0$

$\Rightarrow \quad (n+20)(n-15) = 0$

$\Rightarrow \quad n = -20, 15$

Rejecting the negative value, so we consider

$n = 15$

So, 15 rows need to be there.

COMMON ERR(!)R

In haste, some of the students consider negative value of n. So, please be careful of such type of problems.

Or

Number of seats already put up to the 10th row

$= S_{10}$

$\therefore \quad S_{10} = \frac{10}{2}[2 \times 30 + (10-1)10]$

$= 5(60 + 90) = 750.$

So, the number of seats still required to be put are $1500 - 750 = 750$

3. Given, number of rows $= 17$

Then, the middle row is the $\left(\frac{17+1}{2}\right)$th *i.e.*, 9th row.

$a_9 = a + (9-1)d$

$= a + 8d = 30 + 8 \times 10 = 110$ seats

So, 110 seats will be there in the middle row.

Case Study 6

Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86 m at the Asian Grand Prix in 2017 is the biggest distance for an Indian female athlete.

Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day.

Initially her throw reached 7.56 m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9 cm every week.

During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

Based on the above information, solve the following questions: [CBSE SQP 2023-24]

Q 1. How many throws Sanjitha practiced on 11th day of the camp?

Q 2. What would be Sanjitha's throw distance at the end of 6 weeks?

Or

When will she be able to achieve a throw of 11.16 m?

Q 3. How many throws did she do during the entire camp of 15 days?

Solutions

1. List of the throws during the camp:

$40, (40 + 12), (40 + 2 \times 12), (40 + 3 \times 12), \ldots$

i.e., 40, 52, 64, 76,....

Here the difference of two consecutive terms shows constant *i.e.*, 12.

So, the above sequence forms an AP.

Let 'a' and 'd' be the first term and common difference of an AP respectively.

$$a = 40 \text{ and } d = 52 - 40 = 12$$

On 11th day *i.e.*, $n = 11$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_{11} = 40 + (11-1)(12)$

$= 40 + 10 \times 12$

$= 40 + 120 = 160$

So, 160 throws Sanjitha practiced on 11th day of the camp.

2. List of improve distance in throws by Sanjitha is:

7.56 m, 7.56 m + 9 cm, 7.56 m + 2 × 9 cm,...

i.e., 7.56 m, 7.56 + 0.09 m, 7.56 + 2 × 0.09 m,...

i.e., 7.56, (7.56 + 0.09), (7.56 + 0.18),...

i.e., 7.56, 7.65, 7.74,...

Here the difference of two consecutive terms shows constant *i.e.*, 0.09.

So, the above sequence forms an AP.

Let 'a' and 'd' be the first term and common difference of an AP respectively.

Then, $a = 7.56$ and $d = 7.65 - 7.56 = 0.09$

On 6th week *i.e.*, $n = 6$.

$\because \quad a_n = a + (n-1)d$

$\therefore \quad a_6 = 7.56 + (6-1) \times 0.09$

$= 7.56 + 0.45 = 8.01$

So, Sanjitha's throw distance at the end of 6 weeks is 8.01 m.

Or

Let she will be able to achieve a throw of 11.16 m in n weeks.

Here, first term $(a) = 7.56$, common difference $(d) = 0.09$ and $a_n = 11.16$.

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 11.16 = 7.56 + (n-1) \times (0.09)$

$\Rightarrow 0.09\,(n-1) = 3.6 \quad \Rightarrow \quad n - 1 = \dfrac{3.6}{0.09} = 40$

$\Rightarrow \quad n = 40 + 1 = 41$

So, Sanjitha's will be able to throw 11.16 m in 41 weeks.

3. From list of part (1),

40, 52, 64, 76,...

Here, $a = 40$, $d = 52 - 40 = 12$ and $n = 15$.

$\because \quad S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{15} = \dfrac{15}{2}[2 \times 40 + (15-1)12] = 15(40 + 14 \times 6)$

$= 15\,(40 + 84) = 15 \times 124 = 1860$

So, 1860 throws she do during the entire camp of 15 days.

Very Short Answer Type Questions

Q 1. Write the common difference of the AP:
$\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$.
[NCERT EXEMPLAR; CBSE 2019]

Q 2. Suppose $a = 3, d = -2, l = -11$, find the number of terms exist in an AP.

Q 3. What is the common difference of an AP in which $a_{21} - a_7 = 84$? [CBSE 2017]

Q 4. In an AP, if the common difference (d) is -4 and the seventh term (a_7) is 4, then find the first term. [CBSE 2018]

Q 5. Find the number of terms in the AP: $18, 15\frac{1}{2}, 13, \ldots, -47$. [NCERT EXERCISE; CBSE 2019]

Q 6. How many two digit numbers are divisible by 3? [NCERT EXERCISE; CBSE 2019]

Q 7. Find the 9th term from the end (towards the first term) of the AP: 5, 9, 13, ..., 185. [CBSE 2016]

Q 8. If $a = 2$ and $d = 3$, then find the sum of first 10 terms of an AP.

Q 9. How many terms of AP: 18, 16, 14, ... should be taken so that their sum is zero?

Q 10. If nth term of an AP is $(2n + 3)$, what is the sum of its first five terms?

Short Answer Type-I Questions

Q 1. Find whether −150 is a term of the AP: 17, 12, 7, 2, ... [V.Imp.]

Q 2. Which term of the AP: 8, 14, 20, 26, ... will be 72 more than its 41st term? [CBSE 2017]

Q 3. In an AP, if the sum of third and seventh terms is zero, find its 5th term. [CBSE 2022 Term-II]

Q 4. If 7 times the seventh term of the AP is equal to 5 times the fifth term, then find the value of its 12th term. [CBSE 2022 Term-II]

Q 5. Determine the AP whose third term is 5 and seventh term is 9. [CBSE 2022 Term-II]

Q 6. For what value of n, are the nth terms of two AP's 63, 65, 67, ... and 3, 10, 17, ... equal? [NCERT EXERCISE; CBSE 2017]

Q 7. Determine the AP whose third term is 16 and 7th term exceeds the 5th term by 12. [NCERT EXERCISE; CBSE 2019]

Q 8. Find the middle term of the AP: 6, 13, 20, ..., 216. [CBSE 2015]

Q 9. Find the sum of first 30 terms of AP: −30, −24, −18, ... [CBSE 2022 Term-II]

Q 10. In an AP, if $S_n = n(4n + 1)$, then find the AP. [CBSE 2022 Term-II]

Q 11. Find the sum of first 20 terms of an AP, whose nth term is given as $a_n = 5 - 2n$.

Q 12. In an AP, if $S_5 + S_7 = 167$ and $S_{10} = 235$, then find the AP, where S_n denotes the sum of its first nth terms. [CBSE 2015]

Q 13. How many multiples of 4 lie between 10 and 205? [NCERT EXERCISE; CBSE 2019]

Q 14. How many integers between 200 and 500 are divisible by 8? [CBSE 2017]

Q 15. If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term and common difference. [CBSE 2019]

Short Answer Type-II Questions

Q 1. How many terms are there in AP whose first and fifth terms are –14 and 2, respectively and the last term is 62. [CBSE 2023]

Q 2. Which term of the AP : 65, 61, 57, 53,...... is the first negative term? [CBSE 2023]

Q 3. The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

Q 4. If the 3rd and the 9th terms of an AP are 4 and –8 respectively, which term of this AP is zero? [NCERT EXERCISE; V. Imp.]

Q 5. The sum of the 5th and the 9th terms of an AP is 30. If its 25th term is three times its 8th term, find the AP. [V. Imp.]

Q 6. Each year, a tree grows 5 cm less than it grew the preceding year. If it grew by 1 m in the first year, then in how many years will it have ceased growing? [CBSE 2015]

Q 7. Find the number of terms of the AP: 18, $15\frac{1}{2}$, 13, ..., $-49\frac{1}{2}$ and find the sum of all its terms.

Q 8. The sum of first 15 terms of an AP is 750 and its first term is 15. Find its 20th term. [CBSE 2023]

Q 9. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference. [NCERT EXERCISE; CBSE 2017]

Q 10. How many terms of the AP: 54, 51, 48,... should be taken so that their sum is 513? Explain the double answer.

Q 11. If the sum of first 7 terms of an AP is 49 and that of its first 17 terms is 289, find the sum of first n terms of the AP. [CBSE 2019, 17, 16]

Q 12. Rohan repays his total loan of ₹ 118000 by paying every month starting with the first instalment of ₹ 1000. If he increase the instalment by ₹ 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan has he paid after 30th instalment? [CBSE 2023]

Q 13. Solve the equation for $x : 1 + 4 + 7 + 10 + ... + x = 287$. [NCERT EXEMPLAR; CBSE 2023, 20]

Q 14. Show that the sum of all terms of an AP whose first term is a, the second term is b and the last term is c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$. [NCERT EXEMPLAR; CBSE 2020]

Q 15. Find the sum of all 11 terms of an AP whose middle term is 30. [CBSE 2020]

Long Answer Type Questions

Q 1. The first term of an AP of 20 terms is 2 and its last term is 59. Find its 6th term from the end. [CBSE 2017]

Q 2. Find the number of terms of the AP:
–12, –9, –6,..., 21. If 1 is added to each term of this AP, then find the sum of all terms of the AP thus obtained.

Q 3. If S_n denotes the sum of the first n terms of an AP, prove that $S_{30} = 3(S_{20} - S_{10})$.

Q 4. If the sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms. [CBSE 2023]

Q 5. Find the 60th term of the AP: 8, 10, 12,....., if it has a total of 60 terms and hence find the sum of its last 10 terms. [CBSE 2015]

Q 6. If the sum of the first p terms of an AP is the same as the sum of its first q terms (where $p \neq q$), then show that the sum of first $(p + q)$ terms is zero. [CBSE 2019]

Q 7. In an AP of 50 terms, the sum of the first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the AP. [CBSE 2017; V. Imp.]

Q 8. The ratio of the 11th term to the 8th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms. [CBSE 2023]

Solutions

Very Short Answer Type Questions

1. Given, AP sequence is $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, ...$

or $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}, ...$

Now, common difference,

$$d = \text{Second term} - \text{First term}$$
$$= 2\sqrt{3} - \sqrt{3}$$
$$= \sqrt{3}$$

2. Given, $a = 3$, $d = -2$ and $l = -11$

TR!CK

nth term of an AP is: $a_n = l = a + (n-1)d$.

$\therefore \quad -11 = 3 + (n-1)(-2)$

$\Rightarrow \quad -14 = (n-1)(-2)$

$\Rightarrow \quad 7 = n - 1$

$\Rightarrow \quad n = 8$

Hence, number of terms exist in an AP is 8.

3. Let the first term and common difference of an AP be a and d respectively.

Given, $a_{21} - a_7 = 84$

$\Rightarrow [a + (21-1)d] - [a + (7-1)d] = 84$

[$\because$ nth term of AP, $a_n = a + (n-1)d$]

$\Rightarrow a + 20d - a - 6d = 84$

$\Rightarrow 14d = 84$

$\Rightarrow d = \frac{84}{14} \Rightarrow d = 6$

4. Let 'a' be the first term and 'd' be the common difference of AP.

Given, seventh term $(a_7) = 4$

$\therefore a + (7-1)d = 4$

[$\because$ nth term of AP, $a_n = a + (n-1)d$]

$\Rightarrow a + 6(-4) = 4$ [$\because d = -4$, given]

$\Rightarrow a - 24 = 4$

$\therefore a = 28$

Hence, required first term is 28.

5. The given AP is 18, $15\frac{1}{2}$, 13, ..., -47.

Here, first term $(a) = 18$

and common difference $(d) = 15\frac{1}{2} - 18$

$= \frac{31}{2} - 18 = \frac{31-36}{2} = \frac{-5}{2}$

Suppose there are n terms in the given AP.

Then, $a_n = -47$

$\because$ nth term of AP, $a_n = a + (n-1)d$

$\therefore -47 = 18 + (n-1)\left(\frac{-5}{2}\right)$

$\Rightarrow -47 - 18 = \frac{-5}{2}(n-1)$

$\Rightarrow -65 = \frac{-5}{2}(n-1)$

$\Rightarrow -130 = -5n + 5$

$\Rightarrow -5n = -135$

$\Rightarrow n = \frac{135}{5} = 27$

Hence, there are 27 terms in the given AP.

6. The two-digit numbers divisible by 3 are 12, 15, 18,...,99.

Here, common difference

$= 15 - 12 = 18 - 15 = 3$

So, it forms an AP.

Here, $a = 12$, $d = 3$ and $l = 99$

$\because a_n = l = a + (n-1)d$

$\therefore 99 = 12 + (n-1)3$

$\Rightarrow 3(n-1) = 87$

$\Rightarrow n - 1 = 29 \Rightarrow n = 30$

Hence, the two-digit numbers divisible by 3 are 30.

7.

TR!CK

pth term from the end = (n − p + 1)th term from the beginning, where n is the number of terms.

Given AP is 5, 9, 13, ..., 185.

Here, $a = 5$, $d = 9 - 5 = 4$ and $a_n = 185$.

$\because a_n = a + (n-1)d$

$\therefore 185 = 5 + (n-1)(4)$

$\Rightarrow 180 = 4(n-1) \Rightarrow n - 1 = 45$

$\Rightarrow n = 46$

Now, 9th term from the end = (46 − 9 + 1)th term from the beginning

= 38th term from the beginning

$= a + 37d = 5 + 37 \times 4$ [$\because a_n = a + (n-1)d$]

$= 5 + 148 = 153$

Hence, 9th term from the end is 153.

8. Given, $a = 2$, $d = 3$ and $n = 10$.

TR!CK

The sum of first n terms of an AP is $S_n = \frac{n}{2}[2a + (n-1)d]$.

$\therefore S_{10} = \frac{10}{2}[2 \times 2 + (10-1) \times 3] = 5[4 + 9 \times 3]$

$= 5[4 + 27] = 5 \times 31 = 155$

Hence, sum of first 10 terms of an AP is 155.

9. Let the sum of n terms of an AP be zero, i.e., $S_n = 0$.

Here, $a = 18$ and $d = 16 - 18 = -2$

$\because S_n = \frac{n}{2}[2a + (n-1)d]$ ...(1)

$\therefore 0 = \frac{n}{2}[2 \times 18 + (n-1)(-2)]$ [from eq. (1)]

$\Rightarrow n(36 - 2n + 2) = 0 \Rightarrow 38n - 2n^2 = 0$

$\Rightarrow -2n(n - 19) = 0 \Rightarrow n = 0$ or $n = 19$

But $n = 0$ (not possible)

Hence, the required number of terms is 19.

COMMON ERR(!)R

Some students consider both values of n in the answer but it is wrong approach. So, be careful about this.

10. Given, nth term of an AP is

$T_n = (2n + 3)$

Here, $a = T_1 = 2(1) + 3 = 5$

$l = T_5 = 2(5) + 3 = 13$

TR!CK

The sum of n terms of an AP is $S_n = \frac{n}{2}[a + l]$, *where l is the last term of an AP.*

$\because S_n = \frac{n}{2}(a + l)$

$\therefore S_5 = \frac{5}{2}[5 + 13] = \frac{5}{2} \times 18 = 45$

Hence, sum of first five terms of an AP is 45.

Short Answer Type-I Questions

1. Let nth term of the AP be − 150.

Here, $a = 17$, $d = 12 - 17 = -5$ and $a_n = -150$.

$\because$ nth term of AP, $a_n = a + (n-1)d$

$\therefore -150 = 17 + (n-1)(-5)$

$\Rightarrow -150 = 17 - 5n + 5$

$\Rightarrow \quad 5n = 150 + 22 = 172$

$\Rightarrow \quad n = \frac{172}{5} = 34.4$ (It is not a natural number)

Hence, −150 is not a term of given AP.

COMMON ERR(!)R

Sometimes students consider as round off value of n, i.e., 34 is the answer, but it is wrong. Since, 'n' is always a natural number, so we can't round off the value of n.

2. Given, AP: 8, 14, 20, 26, ...

Here, first term $(a) = 8$

and common difference $(d) = 14 - 8 = 6$

Let its nth term will be 72 more than its 41st term.

$\therefore \quad a_n = a_{41} + 72$

$\Rightarrow \quad a + (n-1)\,d = a + (41-1)\,d + 72$

$[\because a_n = a + (n-1)\,d]$

$\Rightarrow \quad (n-1)\,6 = 40 \times 6 + 72$

$\Rightarrow \quad (n-1)\,6 = 240 + 72$

$\Rightarrow \quad n - 1 = \frac{312}{6} = 52$

$\therefore \quad n = 53$

So, its 53rd term is the required term.

3. Let a and d be the first term and common difference of an AP. The according to the given condition,

$a_3 + a_7 = 0$

TR!CK

nth term of an AP is: $a_n = a + (n-1)d$

$\therefore \quad [a + (3-1)d] + [a + (7-1)d] = 0$

$\Rightarrow \quad a + 2d + a + 6d = 0$

$\Rightarrow \quad 2a + 8d = 0$

$\Rightarrow \quad a + 4d = 0$

$\Rightarrow \quad a + (5-1)\,d = 0$

$\Rightarrow \quad a_5 = 0$

Hence, 5th term is zero.

4. Let a and d be the first term and common difference of an AP. Then,

$7 \times T_7 = 5 \times T_5$

$\therefore \quad 7 \times [a + (7-1)d] = 5 \times [a + (5-1)d]$

$[\because T_n = a + (n-1)d]$

$\Rightarrow \quad 7\,[a + 6d] = 5[a + 4d]$

$\Rightarrow \quad 7a - 5a = 20d - 42d$

$\Rightarrow \quad 2a = -22d \Rightarrow a = -11d$

$\therefore \quad T_{12} = a + (12-1)d$

$= -11d + 11d = 0$

5. Let a and d be the first term and common difference of an AP. Then,

$a_3 = 5$ and $a_7 = 9$

TR!CK

The nth term of an AP is given by $a_n = a + (n-1)d$

$\Rightarrow \quad a + (3-1)d = 5$ and $a + (7-1)d = 9$

$\Rightarrow \quad a + 2d = 5$ and $a + 6d = 9$

On solving, we get $a = 3$ and $d = 1$

TiP

The series of an AP is a, a + d, a + 2d, a + 3d, ...

$\therefore$ The series of an AP is 3, 3 + 1, 3 + 2, 3 + 3, ...

i.e., 3, 4, 5, 6, ...

6. Given, first AP: 63, 65, 67, ...

Here, first term $(a) = 63$

and common difference $(d) = 65 - 63 = 2$

Now, $\quad n$th term, $a_n = a + (n-1)\,d$

$= 63 + (n-1)\,2 = 2n + 61$

Second AP: 3, 10, 17, ...

Here, first term $(a') = 3$

and common difference $(d') = 10 - 3 = 7$

Now, $\quad n$th term, $a'_n = a' + (n-1)d'$

$= 3 + (n-1)\,7 = 7n - 4$

According to the question,

$a_n = a'_n$

$\Rightarrow \quad 2n + 61 = 7n - 4$

$\Rightarrow \quad 5n = 65 \Rightarrow n = 13$

7. Let a be the first term and d be the common difference of given AP.

Given, 3rd term of AP, $a_3 = 16$

TR!CK

nth term of AP, $a_n = a + (n-1)d$, where a and d are the first term and common difference respectively.

$\therefore \quad a + (3-1)d = 16$

$\Rightarrow \quad a + 2d = 16 \quad$ ...(1)

According to the question,

$a_7 = 12 + a_5 \Rightarrow a_7 - a_5 = 12$

$\Rightarrow \quad [a + (7-1)d] - [a + (5-1)d] = 12$

$\Rightarrow \quad (a + 6d) - (a + 4d) = 12$

$\Rightarrow \quad 2d = 12$

$\therefore \quad d = 6$

Put $d = 6$ in eq. (1), we get

$a + 2(6) = 16$

$\Rightarrow \quad a + 12 = 16$

$\therefore \quad a = 4$

Hence, AP will be $a, (a + d), (a + 2d), (a + 3d), ...$

i.e., $\quad 4, (4 + 6), (4 + 2 \times 6), (4 + 3 \times 6), ...$

i.e., $\quad 4, 10, 16, 22, ...$

8. Let a be the first term and d be the common difference of the given AP.

Here, $a = 6$, $d = 13 - 6 = 7$ and $a_n = l = 216$

$\because$ nth term of AP, $a_n = l = a + (n-1)d \quad$ ...(1)

$\Rightarrow \quad 216 = 6 + (n-1)7$

$\Rightarrow \quad 210 = (n-1)7$

$\Rightarrow \quad n - 1 = 30 \Rightarrow n = 31$

$\therefore$ Middle term of the given AP $= \frac{1}{2}(n+1)$

$= \frac{1}{2}(31+1) = \frac{32}{2}$

$= 16$th term

From eq. (1),

$a_{16} = a + (16-1)d$

$= 6 + 15 \times 7 = 6 + 105 = 111$

Hence, the required middle term is 111.

9. Given AP sequence is −30, −24, −18, ...

Let a be the first term and d be the common difference of given AP.

Here $a = -30$, $d = -24 + 30 = 6$

TR!CK

The sum of n terms of an AP is: $S_n = \frac{n}{2}[2a + (n-1)d]$

∴ The sum of 30 terms of an AP is

$$S_{30} = \frac{30}{2}[2 \times (-30) + (30-1)6]$$
$$= 15[-60 + 174] = 15 \times 114 = 1710$$

10. Given, $S_n = n(4n+1) = 4n^2 + n$

TR!CK

nth term of an AP, whose sum is S_n, *is*
$$a_n = S_n - S_{n-1}$$

$$\therefore \quad a_n = 4n^2 + n - [4(n-1)^2 + (n-1)]$$
$$= 4n^2 + n - [4(n^2 + 1 - 2n) + (n-1)]$$
$$= 4n^2 + n - [4n^2 + 4 - 8n + n - 1]$$
$$= 4n^2 + n - [4n^2 - 7n + 3] = 8n - 3$$

∵ The AP series is $a_1, a_2, a_3, \ldots\ldots$

∴ The required AP series is 8(1)−3, 8(2)−3, 8(3)−3, ...

i.e. 5, 13, 21, ...

11. Given, nth term of an AP is

$$a_n = 5 - 2n$$
$$a_1 = 5 - 2(1) = 3$$
$$a_2 = 5 - 2(2) = 1$$
$$a_3 = 5 - 2(3) = -1$$

Here $\quad a = a_1 = 3$

and $\quad d = a_2 - a_1 = 1 - 3 = -2$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{20} = \frac{20}{2}(2 \times (3) + (20-1)(-2))$$
$$= 10(6 + 19 \times (-2)) = 10(6 - 38) = -320$$

12. Let 'a' be the first term and 'd' be the common difference of an AP.

Given, $S_5 + S_7 = 167$

$$\Rightarrow \frac{5}{2}[2a + (5-1)d] + \frac{7}{2}[2a + (7-1)d] = 167$$
$$\left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$
$$\Rightarrow 10a + 5 \times 4d + 14a + 7 \times 6d = 334$$
$$\Rightarrow 24a + 62d = 334$$
$$\Rightarrow 12a + 31d = 167 \quad \ldots(1)$$

Also, $\quad S_{10} = 235$

$$\Rightarrow \frac{10}{2}[2a + (10-1)d] = 235$$
$$\Rightarrow 5(2a + 9d) = 235$$
$$\Rightarrow 2a + 9d = 47 \quad \ldots(2)$$

On multiplying eq. (2) by 6 and subtracting it from eq. (1), we get

$$(12a + 31d) - (12a + 54d) = 167 - 282$$
$$\Rightarrow -23d = -115 \Rightarrow d = 5$$

On putting the value of 'd' in eq.(2), we get

$$2a + 9 \times 5 = 47$$
$$\Rightarrow 2a + 45 = 47 \Rightarrow 2a = 47 - 45 = 2$$
$$\Rightarrow a = 1$$

So, required AP is $a, (a+d), (a+2d), \ldots$

i.e., $1, (1+5), (1+5 \times 2), \ldots$

i.e., 1, 6, 11, ...

13. Let a be the first term and d be the common difference of an AP.

TR!CK

First multiple of 4 which is greater than 10 is 12 and next will be 16 and so on. Therefore, the multiples of 4 are 12, 16, 20, 24,.... .
When we divide 205 by 4, the remainder will be 1. Therefore, 205 − 1 = 204 will be the last number before 205 divisible by 4.

The sequence of multiples of 4 lie between 10 and 205 is as follows:

12, 16, 20, 24, ..., 204

Here, $\quad a = 12, d = 4$ and $a_n = 204$

∵ nth term of AP, $\quad a_n = a + (n-1)d$

$$\therefore \quad 204 = 12 + (n-1)4$$
$$\Rightarrow 204 = 12 + 4n - 4$$
$$\Rightarrow 204 - 8 = 4n$$
$$\Rightarrow 4n = 196$$
$$\Rightarrow n = 49$$

Hence, there are 49 multiples of 4 lies between 10 and 205.

14. Let a be the first term and d be the common difference of an AP.

TR!CK

Clearly, 208 is the first number divisible by 8, lying between 200 and 500. When 500 is divided by 8, then the remainder obtained is 4, so the last number divisible by 8, lying between 200 and 500 is 500 − 4 = 496.

∴ List of integers divisible by 8, lying between 200 and 500 is

208, 216, 224, ..., 496

It represents an AP whose common difference (d) is 8.

Here, first term (a) = 208 and last term (a_n) = 496

∵ nth term of AP, $a_n = a + (n-1)d$

$\therefore \quad 496 = 208 + (n-1)(8)$

$\Rightarrow \quad 496 - 208 = (n-1)\,8$

$\Rightarrow \quad 288 = (n-1)\,8$

$\Rightarrow \quad n-1 = \frac{288}{8} = 36$

$\Rightarrow \quad n = 37$

Therefore, the numbers divisible by 8 lying between 200 and 500 are 37.

COMMON ERROR

Students must read the question very carefully. The question includes "integers between 200 and 500" while, sometimes students take 200 as a first term of the AP because it is also divisible by 8, but it is wrong. So, students take the very first value of this AP as 208.

15. Given, $S_n = 3n^2 - 4n$

TiP

If sum of n terms (S_n) of an AP is given, then nth term (a_n) of the AP can be determined by $a_n = S_n - S_{n-1}$ and common difference by $d = a_n - a_{(n-1)}$.

$\therefore$ nth term is given by

$a_n = S_n - S_{n-1}$

$= 3n^2 - 4n - [3(n-1)^2 - 4(n-1)]$

$= 3n^2 - 4n - [3(n^2 + 1 - 2n) - 4n + 4]$

$= 3n^2 - 4n - 3n^2 - 3 + 6n + 4n - 4$

$= 6n - 7$

and common difference, $d = a_n - a_{(n-1)}$

$= 6n - 7 - [6(n-1) - 7]$

$= 6n - 7 - 6n + 6 + 7 = 6$

Short Answer Type-II Questions

1. Let 'a' be the first term and 'd' be the common difference of an AP.

Given that, first term $(a) = -14$, last term $(l) = 62$ and fifth term, $a_5 = 2$

$\Rightarrow \quad a + (5-1)\,d = 2$

[$\because$ nth term of AP is $a_n = a + (n-1)d$]

$\Rightarrow \quad -14 + 4d = 2$

$\Rightarrow \quad 4d = 16 \Rightarrow d = 4$

Let n terms are in AP.

$\because \quad l = a + (n-1)\,d$

$\therefore \quad 62 = -14 + (n-1)(4)$

$\Rightarrow \quad 4(n-1) = 76 \Rightarrow (n-1) = 19 \Rightarrow n = 20$

So, required number of terms is 20.

2. Let nth term of the AP: 65, 61, 57, 53...... be the first negative term.

Here, first term $(a) = 65$

and common difference $(d) = 61 - 65 = -4$

$\because \quad a_n = a + (n-1)\,d$

$\therefore \quad [a + (n-1)d] < 0$

$\Rightarrow \quad 65 + (n-1)(-4) < 0$

$\Rightarrow \quad (65 - 4n + 4) < 0$

$\Rightarrow \quad (69 - 4n) < 0$

$\Rightarrow \quad (4n - 69) > 0 \Rightarrow n > \frac{69}{4}$

$\Rightarrow \quad n > 17\frac{1}{4}$

$\therefore \quad n = 18$

So, 18th term is the first negative term.

COMMON ERROR

Sometimes students takes the value of $n = 17$ instead of taken the value of $n = 18$.

3. Let 'a' be the first term and 'd' be the common difference of the given AP.

According to the given condition,

$a_{24} = 2 \times a_{10}$

$\Rightarrow \quad a + (24-1)d = 2[a + (10-1)d]$

[$\because$ nth term of the AP, $a_n = a + (n-1)d$]

$\Rightarrow \quad a + 23d = 2 \times (a + 9d)$

$\Rightarrow \quad a + 23d = 2a + 18d$

$\Rightarrow \quad a = 5d$ ...(1)

Now, $a_{72} = a + (72-1)\,d = a + 71d$

$= 5d + 71d$ [from eq. (1)]

$\Rightarrow \quad a_{72} = 76d$ ...(2)

and $\quad a_{15} = a + 14d = 5d + 14d$ [from eq. (1)]

$\Rightarrow \quad a_{15} = 19d$ ...(3)

From eqs. (2) and (3), it is clear that

$a_{72} = 4$ times of a_{15} [$\because 76d = 4 \times 19d$]

Hence proved.

4. Given that $a_3 = 4$ and $a_9 = -8$

Let 'a' be the first term and 'd' be the common difference of the given AP.

$\because$ nth term of AP, $a_n = a + (n-1)d$

$\therefore \quad a_3 = a + (3-1)d$

$\Rightarrow \quad 4 = a + 2d$ ...(1)

and $\quad a_9 = a + (9-1)d$

$\Rightarrow \quad -8 = a + 8d$ ...(2)

On subtracting eq. (1) from eq. (2), we get

$(a + 8d) - (a + 2d) = -8 - 4$

$\Rightarrow \quad 6d = -12$

$\Rightarrow \quad d = -2$

Put $d = -2$ in eq. (1), we get

$a + 2 \times (-2) = 4$

$\Rightarrow \quad a = 8$

Let nth term of this AP be zero.

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 0 = 8 + (n-1)(-2)$

$\Rightarrow \quad 0 = 8 - 2n + 2$

$\Rightarrow \quad 2n = 10$

$\Rightarrow \quad n = 5$

Hence, 5th term of this AP is zero.

5. Let 'a' and 'd' be the first term and common difference respectively of an AP.

According to the given condition,

$a_5 + a_9 = 30$

$\Rightarrow [a + (5-1)d] + [a + (9-1)d] = 30$

[$\because$ nth term of AP, $a_n = a + (n-1)d$]

$\Rightarrow \quad a + 4d + a + 8d = 30$

$\Rightarrow \quad 2a + 12d = 30$

$\Rightarrow \quad a + 6d = 15$ ...(1)

According to the question, $a_{25} = 3 \times a_8$

$\Rightarrow \quad a + (25-1)d = 3 \times [a + (8-1)d]$

$$\Rightarrow \quad a+24d = 3(a+7d)$$
$$\Rightarrow \quad a+24d = 3a+21d$$
$$\Rightarrow \quad 2a = 3d \Rightarrow a = \frac{3}{2}d \quad ...(2)$$

On solving eqs. (1) and (2), we get

$$\frac{3}{2}d + 6d = 15$$
$$\Rightarrow \quad 15d = 30 \Rightarrow d = 2$$
$$\therefore \quad a = \frac{3}{2} \times 2 = 3 \quad \text{[from eq. (2)]}$$

So, required AP is $a, a+d, a+2d, ...$

i.e., $3, 3+2, 3+2\times 2, ...$

i.e., 3, 5, 7, ...

6. Given that tree grows 5 cm or 0.05 m less than preceding year.

TiP

Tree cease growing means the growth of tree becomes zero at some stage.

$\therefore$ The following sequence can be formed:

$1, (1-0.05), (1-2\times 0.05), ..., 0$

i.e., 1, 0.95, 0.90, ..., 0 which is an AP.

Let a and d be the first term and common difference respectively of the given AP.

Here, $a = 1,\ d = 0.95 - 1 = -0.05$ and $a_n = l = 0$.

$$\because \quad l = a_n = a + (n-1)d$$
$$\therefore \quad 0 = 1 + (n-1)(-0.05)$$
$$\Rightarrow \quad 0.05(n-1) = 1$$
$$\Rightarrow \quad n - 1 = \frac{1}{0.05} = \frac{100}{5} = 20$$
$$\Rightarrow \quad n = 20 + 1 = 21$$

Hence, in 21 years, tree will have ceased growing.

7. Given AP is $18, 15\frac{1}{2}, 13, ..., -49\frac{1}{2}$

Let a and d be the first term and common difference respectively of the given AP.

Here, $a = 18,\ d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{-5}{2}$

and $a_n = l = -49\frac{1}{2} = \frac{-99}{2}$

Let the number of terms be n.

$$\because \quad n\text{th term of AP, } a_n = a + (n-1)d$$
$$\Rightarrow \quad \frac{-99}{2} = 18 + (n-1)\left(\frac{-5}{2}\right)$$
$$\Rightarrow \quad -99 = 36 - (n-1)(5)$$
$$\Rightarrow \quad -99 = 36 - 5n + 5$$
$$\Rightarrow \quad 5n = 99 + 41 = 140$$
$$\Rightarrow \quad n = 28$$
$$\therefore \quad S_{28} = \frac{28}{2}\left[2\times 18 + (28-1)\left(\frac{-5}{2}\right)\right]$$

$\left[\because \text{ sum of first } n \text{ terms of an AP, } S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$$= 14\left(36 - 27 \times \frac{5}{2}\right) = 7(72 - 135) = 7(-63)$$
$$= -441$$

Hence, sum of all given terms is -441.

8. Let a and d be the first term and common difference of an AP respectively.

Given, first term $(a) = 15$

$$\therefore \quad S_{15} = 750 \quad \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$$
$$\frac{15}{2}\{2a + (15-1)d\} = 750$$
$$\Rightarrow \quad 2 \times 15 \times 14d = \frac{750 \times 2}{15}$$
$$\Rightarrow \quad 14d = 100 - 30 = 70$$
$$\Rightarrow \quad d = 5$$
$$\therefore \quad a_{20} = a + (20-1)d \quad [\because a_n = a + (n-1)d]$$
$$= 15 + 19 \times 5$$
$$= 15 + 95 = 110$$

So, its required 20th term is 110.

9. Let a and d be the first term and common difference respectively of an AP.

Given, $a = 5,\ l = 45$ and $S_n = 400$

$\because$ Sum of first n terms of an AP,

$$S_n = \frac{n}{2}(a + l)$$
$$400 = \frac{n}{2}(5 + 45) \Rightarrow 400 = \frac{n}{2}(50)$$
$$\Rightarrow \quad n = \frac{800}{50} \Rightarrow n = 16$$
$$\because \quad n\text{th term of AP, } l = a + (n-1)d$$
$$\therefore \quad 45 = 5 + (16-1)d$$
$$\Rightarrow \quad 40 = 15d$$
$$\Rightarrow \quad d = \frac{40}{15} = \frac{8}{3}$$

Hence, the number of terms is 16 and the common difference is $\frac{8}{3}$.

10. Let a and d be the first term and common difference respectively of the given AP.

Here, $a = 54,\ d = 51 - 54 = -3$ and $S_n = 513$.

Let required number of terms be n.

Then, sum of first n terms of the AP.

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\therefore \quad 513 = \frac{n}{2}[2 \times 54 + (n-1)(-3)]$$
$$\Rightarrow \quad 513 \times 2 = n(108 - 3n + 3)$$
$$\Rightarrow \quad 1026 = n(111 - 3n)$$
$$\Rightarrow \quad 1026 = 111n - 3n^2$$
$$\Rightarrow \quad 3n^2 - 111n + 1026 = 0$$
$$\Rightarrow \quad n^2 - 37n + 342 = 0 \quad \text{(divide by 3)}$$
$$\Rightarrow \quad n^2 - 19n - 18n + 342 = 0$$
$$\Rightarrow \quad n(n-19) - 18(n-19) = 0$$
$$\Rightarrow \quad (n-19)(n-18) = 0$$
$$\Rightarrow \quad n - 19 = 0 \text{ or } n - 18 = 0$$
$$\Rightarrow \quad n = 19 \text{ or } n = 18$$

$\therefore$ Sum of 18 terms = Sum of 19 terms

But 19th term is 0.

$[\because a_{19} = 54 + 18 \times (-3) = 54 - 54 = 0]$

Hence, required number of terms is 18.

COMMON ERR(!)R

Some students confused in double answer and make them mistake in writing the answer.

11. Let 'a' and 'd' be the first term and common difference of an AP respectively.

Given, $S_7 = 49$

$\Rightarrow \quad \frac{7}{2}[2a+(7-1)d] = 49$

$\left[\because \text{sum of first } n \text{ terms of an AP, } S_n = \frac{n}{2}\{2a+(n-1)d\}\right]$

$\Rightarrow \quad 2a + 6d = 7 \times 2$

$\Rightarrow \quad a + 3d = 7 \quad ...(1)$

and $\quad S_{17} = 289$

$\Rightarrow \quad \frac{17}{2}[2a+(17-1)d] = 289$

$\Rightarrow \quad 2a + 16d = 17 \times 2$

$\Rightarrow \quad a + 8d = 17 \quad ...(2)$

Subtracting eq. (1) from eq. (2), we get

$(a + 8d) - (a + 3d) = 17 - 7$

$\Rightarrow \quad 5d = 10 \Rightarrow d = 2$

Put $d = 2$ in eq. (1), we get

$a + 3 \times 2 = 7 \quad \Rightarrow \quad a = 1$

Now, $\quad S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)(2)] \quad [\because a = 1 \text{ and } d = 2]$

$= \frac{n}{2}(2 + 2n - 2) = \frac{n}{2} \times 2n = n^2$

Hence, the required sum is n^2.

12. Instalments to be paid by Rohan is 1000, 1100, 1200, ...

Since, the difference between each consecutive terms is 100 (constant). So, this sequence forms an AP.

Let a and d be the first term and common difference of an AP.

$\therefore \quad a = 1000$ and $d = 1100 - 1000 = 100$

$\because \quad a_n = a + (n-1)\,d$

$\therefore \quad a_{30} = 1000 + (30-1)(100)$

$= 1000 + 2900 = 3900$

So, in the 30th instalment, he will pay ₹ 3900.

$\because \quad S_n = \frac{n}{2}[2a + (n-1)\,d]$

$\therefore \quad S_{30} = \frac{30}{2}[2 \times 1000 + (30-1) \times 100]$

$= 15\,(2000 + 2900) = 15 \times 4900$

$= 73500$

So, Rohan has paid ₹ 73500 after 30 instalments.

13. Given, $1 + 4 + 7 + 10 + ... + x = 287 \quad ...(1)$

TiP

First of all check the series $1 + 4 + 7 + ... + x$ is in AP or not.

Let $S_n = 1 + 4 + 7 + 10 + ... + x$

Here, $\quad a_1 = 1, a_2 = 4, a_3 = 7, ...$

Now, $\quad a_2 - a_1 = 4 - 1 = 3$

$a_3 - a_2 = 7 - 4 = 3$

$\because \quad a_2 - a_1 = a_3 - a_2 = 3$

$\therefore$ This series forms an AP with common difference $d = 3$

Let n be the number of terms in that AP.

$\because$ nth term of AP, $a_n = a + (n-1)\,d$

$\therefore \quad x = 1 + (n-1)\,3$

$[\because n\text{th term } a_n = x \text{ and first term } a = a_1 = 1]$

$\Rightarrow \quad n = \frac{x-1}{3} + 1$

$\Rightarrow \quad n = \frac{x+2}{3}$

$\therefore$ Sum of first n terms of an AP,

$S_n = \frac{n}{2}[a_1 + a_n] = \frac{x+2}{2 \times 3}(1+x) = \frac{(x+1)(x+2)}{6}$

Put the value of S_n in eq. (1), we get

$\frac{(x+1)(x+2)}{6} = 287$

$\Rightarrow \quad x^2 + 3x + 2 = 1722$

$\Rightarrow \quad x^2 + 3x - 1720 = 0$

$\Rightarrow \quad x^2 + 43x - 40x - 1720 = 0$

(by splitting the middle term)

$\Rightarrow \quad x(x + 43) - 40(x + 43) = 0$

$\Rightarrow \quad (x + 43)(x - 40) = 0$

$\Rightarrow \quad x = -43, 40$

But x cannot be negative, because at $x = -43$, n is negative, which is not possible.

Thus, required value of x is 40.

COMMON ERR(!)R

Some students make mistake by taking both values of x as answer, but students should be remember that the number of terms n can't be negative.

14. Given, first term $(A) = a$, second term $= b$ and last term $(l) = c$

TR!CK

nth term of AP, $a_n = l = a + (n-1)d$

where, $a_n = l$ = last term of AP and n is the number of terms.

Now, $\quad l = A + (n-1)d$

$\Rightarrow \quad c = a + (n-1)(b-a)$

$(\because \text{ common difference } d = b - a)$

$\Rightarrow \quad \frac{c-a}{b-a} = n - 1$

$\Rightarrow \quad n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a}$

$n = \frac{b+c-2a}{b-a}$

$\because$ Sum of first n terms of an AP is $S_n = \frac{n}{2}(A + l)$

$\therefore \quad S_n = \frac{n}{2}(A+l) = \frac{1}{2}\left(\frac{b+c-2a}{b-a}\right)(a+c)$

$= \frac{(a+c)(b+c-2a)}{2(b-a)}$ **Hence proved.**

15. Given that, the total number of terms in an AP is equal to 11 and the value of the middle most term in an AP is equal to 30.

Now, here $n = 11$ which is odd.

TR!CK

Middle term of series when n is odd $= \left(\frac{n+1}{2}\right)$*th term.*

So, middle term of AP having 11 terms

$= \left(\frac{11+1}{2}\right)$th term $=$ 6th term $= 30$

Let a and d be the first term and common difference respectively of the AP.

$\because$ nth term of an AP is

$a_n = a + (n-1)d$

$\therefore \quad a_6 = a + (6-1)d$

$\Rightarrow \quad 30 = a + 5d$

$\Rightarrow \quad a = 30 - 5d$

$\because$ Sum of first n terms of an AP is

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{11} = \frac{11}{2}[2(30-5d) + (11-1)d]$ $(\because a = 30-5d)$

$= \frac{11}{2}(60) = 330$

Hence, the sum of all 11 terms of an AP is equal to 330.

Long Answer Type Questions

1. Let 'a' and 'd' be the first term and common difference respectively of the given AP.

Given, $a = 2$, $l = 59$ and $n = 20$

$\because$ nth term of AP, $l = a_n = a + (n-1)d$

$\therefore \quad 59 = 2 + (20-1)d$

$\Rightarrow \quad 59 - 2 = 19d \Rightarrow 19d = 57$

$\Rightarrow \quad d = \frac{57}{19} = 3$

$\because$ nth term from the end $= l - (n-1)d$

$\therefore$ 6th term from the end

$= 59 - (6-1)3 = 59 - 15 = 44$

Hence, the 6th term from the end is 44.

2. Given AP is $-12, -9, -6, \ldots, 21$.

Let a and d be the first term and common difference respectively of the AP.

Here, $a = -12$, $d = -9 - (-12) = 3$ and $l = 21 = a_n$

Let n be the number of terms in the given AP.

$\because$ nth term of AP, $l = a_n = a + (n-1)d$

$\therefore \quad 21 = -12 + (n-1)(3)$

$\Rightarrow \quad 21 = -12 + 3n - 3$

$\Rightarrow \quad 21 + 15 = 3n$

$\Rightarrow \quad n = \frac{36}{3}$

or $\quad n = 12$

On adding 1 to each term in given AP, new AP so formed is $-11, -8, -5, \ldots, 22$

Here, $a = -11$, $d = -8 - (-11) = 3$, $n = 12$ and $l = 22$

$\because$ Sum of n terms of the AP, $S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_n = \frac{12}{2}(-11 + 22) = 6 \times 11 = 66$

Hence, the required number of terms is 12 and sum of all terms of new AP is 66.

3. Let the first term and common difference of the given AP be 'a' and 'd' respectively.

$\because$ Sum of first n terms of the AP,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{30} = \frac{30}{2}[2a + (30-1)d]$

$\Rightarrow \quad S_{30} = 15(2a + 29d) = 30a + 435d$ ...(1)

$S_{20} = \frac{20}{2}[2a + (20-1)d]$

$\Rightarrow \quad S_{20} = 10(2a + 19d) = 20a + 190d$ ...(2)

and $\quad S_{10} = \frac{10}{2}[2a + (10-1)d]$

$\Rightarrow \quad S_{10} = 5(2a + 9d) = 10a + 45d$ ...(3)

Now, $3(S_{20} - S_{10}) = 3[(20a + 190d) - (10a + 45d)]$

[from eqs. (2) and (3)]

$= 3[10a + 145d]$

$= 30a + 435d = S_{30}$ [from eq. (1)]

Hence proved.

4. Let a and d be the first term and common difference respectively of the given AP.

Given, $S_6 = 36$ and $S_{16} = 256$

TR!CK

Sum of first n terms of an AP is: $S_n = \frac{n}{2}[2a + (n-1)d]$.

$\Rightarrow \quad \frac{6}{2}[2a + (6-1)d] = 36$

and $\quad \frac{16}{2}[2a + (16-1)d] = 256$

$\Rightarrow \quad 3[2a + 5d] = 36$ and $8[2a + 15d] = 256$

$\Rightarrow \quad 2a + 5d = 12$ ...(1)

and $\quad 2a + 15d = 32$ ...(2)

On subtracting eq. (1) from eq. (2), we get

$(2a + 15d) - (2a + 5d) = 32 - 12$

$\Rightarrow \quad 10d = 20 \Rightarrow d = 2$

Put $d = 2$ in eq. (1), we get

$2a + 5 \times 2 = 12$

$\Rightarrow \quad 2a = 12 - 10 = 2 \Rightarrow a = 1$

Now, $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1)2] = \frac{n}{2}[2 + 2n - 2]$

$= \frac{n}{2}[2n] = n^2$

$\therefore \quad S_{10} = (10)^2 = 100$

Hence, sum of first 10 terms is 100.

5. Given AP is 8, 10, 12, ...

Let a and d be the first term and common difference of the AP respectively.

Here, $a = 8$ and $d = 10 - 8 = 2$.

Let the number of terms in the given AP be n.

$\because$ nth term of AP, $a_n = a + (n-1)d$

$\therefore \quad a_{60} = 8 + (60-1)\,2 = 8 + 59 \times 2 = 8 + 118 = 126$

or $a_{60} = 126$

Hence, 60th term is 126.

Now, sum of its last 10 terms $= S_{60} - S_{50}$

$= \frac{60}{2}[2 \times 8 + (60-1) \times 2] - \frac{50}{2}[2 \times 8 + (50-1) \times 2]$

$\left[\because \text{sum of first } n \text{ terms of an AP, } S_n = \frac{n}{2}(2a + (n-1)d)\right]$

$= 30(16 + 59 \times 2) - 25(16 + 49 \times 2)$

$= 30(16 + 118) - 25(16 + 98)$

$= 30 \times 134 - 25 \times 114 = 4020 - 2850 = 1170$

Hence, sum of last 10 terms of given AP is 1170.

6. Let a and d be the first term and common difference of the AP respectively.

According to the question, $S_p = S_q$

TR!CK

Sum of first n terms of an AP, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$

$\Rightarrow \quad p(2a + pd - d) = q(2a + qd - d)$

$\Rightarrow \quad 2ap + p^2d - pd = 2aq + q^2d - qd$

$\Rightarrow \quad 2a(p-q) + d(p^2 - q^2) - d(p-q) = 0$

$\Rightarrow \quad (p-q)\,[2a + d(p+q) - d] = 0$

$\Rightarrow \quad 2a + (p+q-1)\,d = 0 \; (\because p \neq q)$ ...(1)

Now, sum of first $(p+q)$ terms $= S_{p+q}$

$= \frac{(p \;\; q)}{2}\,[2a + (p+q-1)d]$

$= \left(\frac{p \;\; q}{}\right) \times$ [from eq. (1)]

$= 0$ **Hence proved.**

7.

TiP

Calculate 36th and 50th terms of AP as consider first term and last term respectively, because the sum of last 15 terms of AP is given and there is no idea about 'a' and 'd' of given AP.

Let the first term and common difference of an AP be 'a' and 'd' respectively.

Given, number of terms in this AP, $n = 50$

Sum of first 10 terms of this AP, $S_{10} = 210$

$\Rightarrow \frac{10}{2}[2a + (10-1)d] = 210$

$\left[\because \text{sum of first } n \text{ terms of an AP, } S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow \quad 2a + 9d = 42$ ...(1)

Now, 36th term of this AP,

$a_{36} = a + (36-1)d = a + 35d$

$[\because \text{nth term of an AP; } a_n = a + (n-1)\,d]$

and 50th term of this AP, $a_{50} = a + 49d$

$\because$ Sum of last 15 terms of this AP $= 2565$

$\therefore \quad \frac{15}{2}(a_{36} + a_{50}) = 2565 \quad \left[\because S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow \frac{15}{2}(a + 35d + a + 49d) = 2565$

$\Rightarrow \quad 2a + 84\,d = 171 \times 2$

$\Rightarrow \quad a + 42d = 171$ ...(2)

Put the value of 'a' from eq. (2), in eq. (1), we get

$2(171 - 42d) + 9d = 42$

$\Rightarrow \quad 75d = 300 \quad \Rightarrow \quad d = 4$

Put the value of 'd' in eq. (2), we get

$a + 168 = 171 \quad \Rightarrow \quad a = 3$

So, required AP is:

$a, a+d, a+2d, a+3d, \ldots$

$\therefore \quad 3, 3+4, 3+2 \times 4, 3+3 \times 4, \ldots$

or $\quad 3, 7, 11, 15, \ldots$

8. Let a and d be the first term and common difference of an AP respectively.

$\because$ nth term of AP, $a_n = a\,(n-1)d$

$\therefore$ 11th term of AP, $a_{11} = a + (11-1)\,d = a + 10d$

and 18th term of AP, $a_{18} = a + (18-1)d = a + 17d$

Given, $\frac{a_{11}}{a_{18}} = \frac{2}{3} \quad \Rightarrow \quad \frac{a + 10d}{a + 17d} = \frac{2}{3}$

$\Rightarrow \quad 3a + 30d = 2a + 34d$

$\Rightarrow \quad a = 4d$ ... (1)

$\therefore \quad \frac{\text{5th term of AP}}{\text{21st term of AP}} = \frac{a + (5-1)d}{a + (21-1)d} = \frac{a + 4d}{a + 30d}$ [from eq. (1)]

$= \frac{4d + 4d}{4d + 20d} = \frac{8d}{24d} = \frac{1}{3}$

$\therefore$ Sum of first n terms of AP is

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_5 = \frac{5}{2}[2a + (5-1)d] = \frac{5}{2}[2 \times 4d \times 4d]$

$= \frac{5}{2}(8d + 4d) = \frac{5}{2} \times 12d = 30d$ [from eq. (1)]

and $S_{21} = \frac{21}{2}[2a + (21-1)d] = \frac{21}{2}[2 \times 4d + 20d]$

$= \frac{21}{2}(8d + 20d) = \frac{21}{2} \times 28d = 294d$ [from eq. (i)]

$\therefore \frac{S_5}{S_{21}} = \frac{30d}{294d} = \frac{10}{98} = \frac{5}{49}$

Hence, required ratios are $a_5 : a_{21} = 3 : 1$ and $S_5 : S_{21} = 5 : 49$

Chapter Test

Multiple Choice Questions

Q 1. If 7th term and 13th term of an AP are 34 and 64 respectively, then its 18th term is:

a. 89 b. 90 c. 92 d. 94

Q 2. If the sum of n terms of an AP is $3n^2 + n$ and its common difference is 6, then its first term is:

a. 2 b. 4 c. 5 d. 6

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): The common difference of an AP in which $a_{15} - a_{10} = 30$ is 6.

Reason (R): The nth term of the sequence 8, 13, 18,........ is $5n + 3$.

Q 4. Assertion (A): The sum of the series with the nth term $T_n = 4 - 2n$ is -208, when number of terms is 16.

Reason (R): The sum of AP series is determined by $S_n = \frac{n}{2}[2a + (n-1)d]$.

Fill in the Blanks

Q 5. In any arithmetic progression, if each term is increased by 3, then the new progression series is formed ....

Q 6. If the common difference is .... then each term of the AP will be same as the first term of the AP.

True/False

Q 7. If nth term of an AP is a_n, then the common difference is determined by $d = a_n - a_{n-1}$.

Q 8. A sequence follow certain rule is a progression.

Case Study Based Question

Q 9. There is a great demands of electrical appliances (*i.e.*, Freeze, Television, Cooler etc.) an electrical appliance manufacturing company decided to increase its production. In every five years, the company doubles its increased production. Following the same process of increasing production, the production of company in its 5th year was 10000 sets, in the 6th year it was 11000 sets and so on.

Based on the above information, solve the following questions:

(i) Find the production of the company during first year.

(ii) In which year, the production is 20,000 sets?

Or

Find the sum of production during first 9 yr.

(iii) In how many years, company produce 2,16,000 sets?

Very Short Answer Type Questions

Q 10. Find the 6th term from the end of the AP: 17, 14, 11,......, –40.

Q 11. Find the sum of 20 terms of the AP: 1, 4, 7, 10,..... .

Short Answer Type-I Questions

Q 12. Which term of the arithmetic progression 5, 15, 25,....... will be 130 more than its 31st term?

Short Answer Type-II Questions

Q 13. A man repays a loan of ₹3250 by paying ₹20 in the first month and then increases the payment by ₹15 every month. How long will it take him to clear the loan?

Long Answer Type Question

Q 14. The ratio of the sums of m and n terms of an AP is $m^2 : n^2$. Show that the ratio of the mth and nth terms is $(2m - 1) : (2n - 1)$.